

Lobe Overflow as the Likely Cause of Pericenter Outburst in an SMBH Orbiter

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ABSTRACT

A very large lobe overflow event is suggested to explain the $0.^m4$ brightening observed in K band at pericenter passage of the star known as S2 that orbits the Galaxy's supermassive black hole (SMBH). Known observed properties of S2 that contribute to lobe filling are 1) the enormous mass ratio, M_{SMBH}/M_{S2} , 2) S2's fast rotation, and 3) S2's large orbital eccentricity. Published estimates have given limiting lobe sizes of order 100 to 300 R_{\odot} but, with S2's fast rotation taken into account, the computed lobe size is *much* smaller, being compatible with either a main sequence OB star or a stripped evolved star. An important evolutionary consideration that predicts very large pericenter overflows is envelope expansion following mass loss that is characteristic of highly evolved stars. Material removed by lobe overflow at pericenter is replenished by envelope expansion as an evolved star awaits its next pericenter passage. An observational signature of lobe overflow for upcoming pericenter passages would be appearance of emission lines as the ejected gas expands and becomes optically thin.

Subject headings: Galaxy: center — stars: individual

1. Background

Prospects for understanding the evolution of the Galactic center region were boosted by discovery and quantitative investigation (Schödel, et al. 2002) of a star designated as S2 that is bound to the Galaxy's supermassive black hole (SMBH) in a 15.8 year orbit, with additional stars in the region under similar investigation – see also Ghez, et al. (2003a). Many papers have since added observations and ideas on individual SMBH orbiters and

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on the statistics and collective properties of stars in the inner Galactic center region, e.g. Ghez, et al. (2003b,c); Alexander & Morris (2003); Blum, et al. (2003); Eisenhauer, et al. (2005); Gillessen, et al. (2009); Davies & King (2005); Gillessen, et al. (2013); Witzel, et al. (2014) and an extensive review by Genzel, et al. (2010).

Issues abound regarding the evolutionary states of these stars and how they came to be in the inner Galactic center, e.g. Davies & King (2005); Zhang, et al. (2013); Gillessen, et al. (2009); Genzel, et al. (2010). Stars arriving on nearly parabolic or even hyperbolic orbits could suffer major stripping on initial pericenter passage (Davies & King 2005), with the lost material carrying away enough orbital energy to leave the remnant in an elliptical orbit, filling its limiting lobe at pericenter. Alternatively, a star that has been trapped into a tight orbit while well detached from its lobe could later undergo evolutionary expansion and attain lobe filling. Perhaps a lost binary companion may carry off the requisite orbital energy at first encounter with the SMBH and leave the remaining star bound.

Eisenhauer, et al. (2005) found most of the brighter inner orbiters to be B0 to B9 main sequence stars, with S2 in the range O8 to B0, and with rotation velocities typical of B stars in the Galactic disk. Davies & King (2005) argued that they are actually tidally stripped remnants of AGB stars that now superficially resemble main sequence stars and estimated S2’s mass at below a solar mass, specifically about $0.8 M_{\odot}$. All in all, mass estimates for S2 that have been published or correspond to observed (main sequence) spectral types range from 0.8 to more than $20 M_{\odot}$. Accordingly, we simply adopt $10 M_{\odot}$ for exploratory lobe size computations.

2. Brightening of Star S2 Due to Lobe Overflow

S2 has continued as the most thoroughly discussed SMBH orbiter, largely due to its having been observed over more than a full orbit, and having brightened by about $0.^m40$ in K band, coincident with pericenter passage (Gillessen, et al. 2009). Gillessen, et al. offered seven ideas for explaining the brightening, however they then ruled out four of the ideas and argued against likelihood of the other three. Consequences of lobe overflow – a very common player in a variety of close binary issues – were not among the seven ideas. Lobe overflow might not be considered if one were thinking only of the huge orbital scale (of order 1000 AU) compared to the size of a main sequence star, but it turns out that S2’s limiting lobe is actually similar in size to a main sequence star of, say, $10 M_{\odot}$, as shown in §4. Lobe overflow is an attractive idea for S2 brightening – one could postulate that S2 exceeded its limiting lobe at the 2002 pericenter passage and ejected a strong puff of material that quickly expanded in vacuum so as to appear as a rather large cloud of brightly emitting gas. The

lobe size issue will now be addressed.

3. Lobe Size Essentials

To place the present work in context, we review the formal relations in estimates for (1) tidal radii and (2) limiting lobes. Although these two terms are quite distinct, both often go by the name ‘Roche limit’ and some recent papers treat them as equivalent, thereby leading to considerable confusion and perhaps even wrong conclusions. Item 1, tidal radius, concerns the distance from a mass at which an idealized fluid mass (usually a small satellite) is disrupted when tidal stretching matches or exceeds the satellite’s cohesion due to self-gravity. The simple tidal radius concept considers test particles on the surface of a self-gravitating sphere of mass m and radius r at a distance d from an object of mass M . The test particles, located on opposite ends of a diameter of m on the line of centers, suffer a stretching force (surface to center) per unit particle mass due to external mass M of $2GMr/d^3$, assuming r to be very small. The effective compressional force per unit particle mass between surface and center that can result in a static configuration is the object’s surface gravity, Gm/r^2 . Quantity d is the tidal radius and marks the distance from M at which a very small object is disrupted, so the final relation pertains to the test particles being arbitrarily close together. If r is not small compared to d , then the simple relation may still give the tidal radius approximately, with only gravitation considered, although the full relation is then more complicated. A flaw in this picture with regard to stretched objects of finite size is that the satellite is presumed spherical, whereas tidally stretched stars lack front-to-back symmetry (i.e. have “teardrop” shapes). But more important for the case of star S2 and probably other SMBH orbiters is that rotation of m is not considered in the traditional tidal limit development. In summation, the tidal limit relation between d and r , $d = r(2M/m)^{1/3}$, can be inverted with d set to the orbital separation to give roughly correct limiting size where the assumptions apply, namely *for small, synchronously rotating¹ satellites*, but may be wrong by orders of magnitude for fast rotating stars such as S2.

Item 2 is commonly called a Roche lobe, although it was not originated or even considered by Roche, who did however consider a special case of the potential utilized today. As the idea is to specify a size limit set by the condition that material not be spilled from a star, the descriptive term ‘limiting lobe’ serves well. The insight that spawned the concept came

¹In this context, ‘synchronous rotation’ means that angular star rotation and mean orbital angular rotation are equal or, equivalently, that the star rotates once per orbit period. Note that there are other meanings of ‘synchronous rotation’, each valid in its own context.

from Kuiper (1941), who realized that tidal force is an unnecessary complication with regard to the lobe size limit, as only one point, not two, need be considered, and only ordinary effective gravity at that point, not differences between two points, need be computed. The procedure is (step 1) to find the point along the line of centers where ordinary gravitational (not differential tide raising) forces due to M and m , along with local rotational force, add to zero. Material that is stationary in a frame that co-rotates with the star is not bound to the star at this special point, so an ejection nozzle forms. Asynchronous examples are now treated via a factor F^2 (see §4) that alters the centrifugal term without affecting the basic idea of locating the effective gravity null point (Plavec 1958; Limber 1963). A definite equipotential that defines the star surface passes through the special point of null effective gravity, so (step 2) numerically integrate the volume, V_{lobe} , enclosed by that equipotential and thereby find the equivalent-sphere mean radius as

$$R_{mean} = (3V_{lobe}/4\pi)^{1/3}. \quad (1)$$

Kuiper assumed synchronous rotation, which is the expected and observationally indicated case for very close binaries (due to tidal locking).

Conditions that lead to small limiting lobes for SMBH orbiters are the enormous mass ratio, large orbital eccentricity, and – not previously emphasized in the literature – fast rotation. For fixed SMBH mass², lobe size decreases with decreasing star mass, decreasing orbit size, increasing eccentricity, and increasing star rotation. S2 is known to be a fast rotator, while any kind of tidal locking would produce exceedingly slow rotation in view of the 15.8 year (Ghez, et al. 2003c) orbit period, P_{orb} . The fastest locked rotations would be for locking to the pericenter orbital angular rate and give P_{rot} around half a year for S2, whereas the *measured* $V_{rot} \sin i$ is $220 \pm 40 \text{ km sec}^{-1}$ (Ghez, et al. 2003c), so there is no tidal locking of any kind. Whether the star’s equator is aligned with the orbit plane is not known but the orbital $\sin i$ is 0.7040 ± 0.0058 (Gillessen, et al. 2009) so, under the assumption of alignment, $V_{rot} \approx 312 \pm 57 \text{ km sec}^{-1}$. The corresponding angular rotation, assuming $R_{eq} = 5.0R_{\odot}$, is ≈ 7100 times the mean orbital angular rate, and rotational force becomes important in setting local effective gravity. It will be shown below that the problem of main sequence SMBH orbiters being too small to exceed their limiting lobes can disappear if they rotate at typical B star rates, as does S2.

²We adopt $M_{SMBH} = 4.31 \times 10^6 M_{\odot}$ (Gillessen, et al. 2009) and thus a mass ratio, M_{SMBH}/M_{10} , of 4.31×10^5 for a $10 M_{\odot}$ star.

3.1. The Eggleton Approximation

Approximation formulas are often used to estimate lobe size, most commonly one by Eggleton (1983) that reproduces accurately computed mean lobe radii, based on the Kuiper logic, to better than 1 percent over the full range of mass ratio, from 0 to ∞ ³. Although a rather small computer program can generate such lobe radii with negligible error, and some public binary star programs list lobe radii as incidental output, the Eggleton approximation has provided a one-line lobe calculation in many evolutionary programs where 1 percent accuracy may be sufficient. However note that the Eggleton formula is specifically for synchronous rotation and not meant for stars that rotate faster or slower than synchronously. It will give limiting lobe radii that are too large by orders of magnitude if applied to fast rotators such as star S2, and may be responsible for misleading conclusions where rotation rates are unknown. An algorithm that follows the Kuiper logic, enhanced to handle arbitrary rotation and eccentricity, is not difficult to program and avoids the approximations of fitted formulas. Most *accurate* limiting lobe computations now adopt Kuiper’s strategy, usually via one of the commonly used binary system light/velocity curve programs, although another option can be the collection of intricate approximation formulas by Sepinski, et al. (2007) that account for asynchronism and eccentricity.

4. Quantitative Estimate of S2’s Lobe Size

Computation of a binary component’s limiting lobe geometry begins with solution for a point of null effective gravity along the line of star centers (x-axis), thereby locating the nozzle from which matter flows if the lobe is filled or slightly overfilled. The relevant equation for the S2 problem must account for orbital eccentricity and the star’s rotation in addition to the gravity of both objects, as does eqn. 3 of Wilson (1979) for the derivative of potential⁴ in the x-direction, which is zero at the null point. That equation is

³Incidentally, we checked the 1 percent accuracy statement in Eggleton (1983) at the request of Prof. Eggleton, finding no discrepancies as large as 0.8 percent among 18 widely spread mass ratios, of which only two exceeded half a percent. Column 2 of Eggleton’s Table 1 (mean lobe radii from integrated volumes) was reproduced to all printed digits except for two differences of 1 in the last place. Column 6, the Eggleton approximation, was reproduced exactly. The checks were done with the WD (Wilson & Devinney 1971) computer model.

⁴The potential is a modified version according to the convention in Kopal (1959).

$$\frac{d\Omega}{dx} = -\frac{x}{(x^2 + y^2 + z^2)^{3/2}} + \frac{q(D - x)}{([D - x]^2 + y^2 + z^2)^{3/2}} + F^2(1 + q)x - q/D^2. \quad (2)$$

Rotation enters via a parameter F , the ratio of rotational angular velocity ω_{rot} to mean (i.e. time-averaged) orbital angular velocity ω_{orb} . Other input quantities are the component mass ratio ($q = M2/M1$), momentary separation of star centers (D), and x, y, z rectangular coordinates of a point at which $d\Omega/dx$, and subsequently Ω , are to be computed. Here S2 is taken to be star 1 and the SMBH is object 2, so the mass ratio is a large number rather than its reciprocal. The unit for x, y , and z is D while D is in unit a , the semi-major axis of the relative orbit, in computations with equation 2, with $D = 1 - e$ at periastron or pericenter. The location of the null point along the line of centers is found by setting $y = z = 0$ and $d\Omega/dx$ also to 0, setting the dimensionless angular rotation F and eccentricity e to values of interest, and then solving for x by numerical inversion (such as Newton-Raphson iteration). The potential at the null point then establishes the lobe surface's 3-dimensional form as an equipotential that includes the null point (see eqn. 1 of Wilson (1979) for the generalized defining potential). The equipotential's enclosed volume (V) can then be integrated numerically via the defining equation and a mean lobe radius found from eqn. 1. A final step computes equatorial rotation velocity, V_{eq} , from angular velocity. That calculation is simplified by the star being almost axially symmetric and its equator circular at these fast rotation rates, so there is no issue of where along the equator the result applies. Accordingly

$$V_{eq} = R_{eq}\omega_{orb}F, \quad (3)$$

with length in km, time in seconds, and mean orbital angular velocity, $\omega_{orb} = 2\pi/P_{orb}$, in *radians/sec*.

The binary star modeling and analysis program (WD program⁵) applied here has refinements that allow reliable operation in difficult circumstances. For example, its Newton-Raphson iterations (for inversion of equation 2 to find the effective gravity null point) evaluate several Taylor series terms beyond the usual first derivative term. This point is mentioned so that readers who may write their own inversion program to check our results are not disappointed by failed computations. A relatively simple inversion scheme can converge well for ordinary mass ratios but not for ultra-large mass ratios such as the 4.31×10^5 of the

⁵The WD program's most recent public version, with documentation and sample input files, can be downloaded from anonymous FTP site <ftp.astro.ufl.edu>. Go to sub-directory <pub/wilson/lcdc2013>.

present problem. Also important for fractionally tiny lobes (large q , large F) is to begin iterations already close to the null point, so as to avoid an initial jump *beyond* the proper range between the star centers, from which recovery is difficult. Fortunately such a configuration admits particularly good starting estimates of the null point’s location. To see this readily, write eqn. 2 as it applies along the line of centers at the null point,

$$0 = -\frac{1}{x^2} + \frac{q}{(D-x)^2} + F^2(1+q)x - \frac{q}{D^2}. \quad (4)$$

This form is a quintic equation in x , soluble only iteratively, but with x very small the second and fourth terms on the right side very nearly cancel so that the remaining terms (also replacing $1+q$ with the very large q) lead to a simple result,

$$x \approx F^{-2/3} q^{-1/3}. \quad (5)$$

The approximation is reasonably accurate only for quite small x , although very accurate for SMBH orbiters and perhaps usefully accurate for M_2/M_1 of a few hundred or more.

Inputs to the lobe size computation for S2 were $e = 0.88$ (Gillessen, et al. 2009) and $M_{SMBH}/M_{S2} = 4.31 \times 10^5$ (mass ratio), along with a few well spaced F ’s. One of the F ’s is close to the nominal value of 7100 that goes with our rough estimate of V_{eq} that assumed alignment of the equatorial and orbit planes in §3. The resulting mean lobe radius is $6.5R_\odot$, which is larger than a $10 M_\odot$ main sequence star (about $3 R_\odot$ on the ZAMS to $5 R_\odot$ at the TAMS), although the spectral type estimate by Eisenhauer, et al. (2005) extends to O8, for which a main sequence radius can exceed $6.5R_\odot$. A stripped highly evolved star that *resembles* a main sequence star, as in Davies & King (2005), remains a candidate. With either kind of star, the idea of lobe overflow at pericenter passage now becomes a real possibility. Table 1 has mean lobe radii⁶ for four assumed angular rotation velocities (F ’s) of the $10 M_\odot$ model orbiter to give a sense of how steeply lobe size depends on rotation rate. A check to see if the program gives the right order of lobe size is provided by calculation of the equatorial radius of a $10 M_\odot$ isolated star (no SMBH) that is marginally unbound at the equator while rotating at one of the table values, 307 km sec^{-1} . If the magnitudes of rotational and gravitational force are then equated, the equatorial radius will be given by $R_{eq} = GM/V^2$, which evaluates to $20.2R_\odot$ for a $10 M_\odot$ star. The corresponding *mean* radius will be smaller since R_{pole} is smaller than R_{eq} , so rotation alone produces a limiting size only about three times greater than do the combined effects of rotation and the SMBH gravity. The purely

⁶Note that these are ‘equivalent sphere’ radii, not distances to the effective gravity null point.

gravitational lobe radius for a slowly rotating star, with $e = 0.88$ and the present problem’s adopted masses, is $\approx 100R_{\odot}$, so the effect of fast rotation on lobe size is not small.

4.1. Why Such Large Scale Ejection?

A remaining issue is why a *huge* puff would be ejected at pericenter passage. The ordinary context of lobe overflow is the synchronous-circular case that is commonly encountered in close binary systems, where gas leaks out quiescently and is usually difficult or impossible to detect photometrically. S2, being a very fast rotator, will not undergo the gentle process of the synchronous-circular case with its low ejection velocity. The supersynchronous case is very different, with an ejection velocity close to the star’s equatorial velocity, which is of order 300 km sec^{-1} for our model of S2. And why would a large amount of gas be ejected? Suppose the Davies & King (2005) proposal, that the close-in orbiters are tidally stripped highly evolved stars, is correct, and that S2 is typical. Well known (e.g. Plavec (1968)) is that radii of highly evolved (i.e. chemically stratified) stars increase with loss of envelope matter, in contrast with shrinkage for unevolved and modestly evolved stars. S2 has 15.8 years between pericenter passages to expand following each pass and could arrive at pericenter not just marginally filling its lobe but substantially overfilling it. Although a quantitative estimate of the overfilling will require reasonably good estimates of S2’s internal structure that are not now in hand, the qualitative picture is that S2 may reach pericenter ready to send very fast moving gas through a large open nozzle, leading to a very large ejection event. One test of this idea, waiting for the next pericenter passage, is that emission lines should appear as the ejected gas expands and becomes optically thin. Naturally some or all of these expectations may be anticipated as other SMBH orbiters pass through their pericenters.

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Table 1. Dependence of 'Equivalent Spherical Volume' Lobe Radius on Equatorial Velocity for a $10 M_{\odot}$, $e = 0.88$ SMBH Orbiter Representing S2

F (rotation parameter)	Equatorial Velocity ($km \cdot sec^{-1}$)	'Equivalent Sphere' Lobe Size (R_{\odot})
1000	44	23.6
3000	131	11.4
7000	307	6.5
10000	438	5.1